1. For a group of 7 people, find the probability that all 4 seasons (winter, spring, summer, fall) occur at least once each among their birthdays, assuming that all seasons are equally likely.

ANSWER:

Let Ai be the event that there are no birthdays in the ith season.

The probability that all seasons occur at least once is 1 P(A1 [ A2 [ A3 [ A4).

Note that A1 \ A2 \ A3 \ A4 = ;. Using the inclusion-exclusion principle and the symmetry of the seasons, P(A1 [ A2 [ A3 [ A4) = X 4 i=1 P(Ai) X 3 i=1 X j>i P(Ai \ Aj ) + X 3 i=1 X j>i X k>j P(Ai \ Aj \ Ak) = 4P(A1) 6P(A1 \ A2)+4P(A1 \ A2 \ A3).

We have P(A1) = (3/4)7. Similarly, P(A1 \ A2) = 1 27 and P(A1 \ A2 \ A3) = 1 47 . Therefore, P(A1 [ A2 [ A3 [ A4) = 4( 3 4 )7 6 27 + 4 47 .

So the probability that all 4 seasons occur at least once is 1 4( 3 4 )7 6 27 + 4 47 ⇡ 0.513.

1. Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday? (This problem can be done either directly using the naive definition of probability, or using inclusion-exclusion.)

ANSWER:

Direct Method:

There are two general ways that Alice can have class every day: either she has 2 days with 2 classes and 3 days with 1 class, or she has 1day with 3 classes, and has 1 class on each of the other 4 days.

The number of possibilities for the former is 5 2 6 2 2 63 (choose the 2 days when she has 2 classes, and then select 2 classes on those days and 1 class for the other days).

The number of possibilities for the latter is 5 1 6 3 64.

So the probability is 5 2 6 2 2 63 + 5 1 6 3 64 30 7 = 114 377 ⇡ 0.302.

